Exercise 22

Find the intersection of the two planes with equations $3(x-1)+2y+(z+1)=0$ and $(x-1)+4y-(z+1)=0$.

Solution

The intersection for two planes is a straight line, which can be parameterized as

$$\mathbf{y}(t)=\mathbf{m}t+\mathbf{b},$$

where $\mathbf{m}$ is the direction vector and $\mathbf{b}$ is the position vector for any point on the line. The normal vectors to the given planes are obtained from the coefficients of x, y, and z: $(3,2,1)$ and $(1,4,-1)$. Take the cross product of these two to find the direction vector of the line.

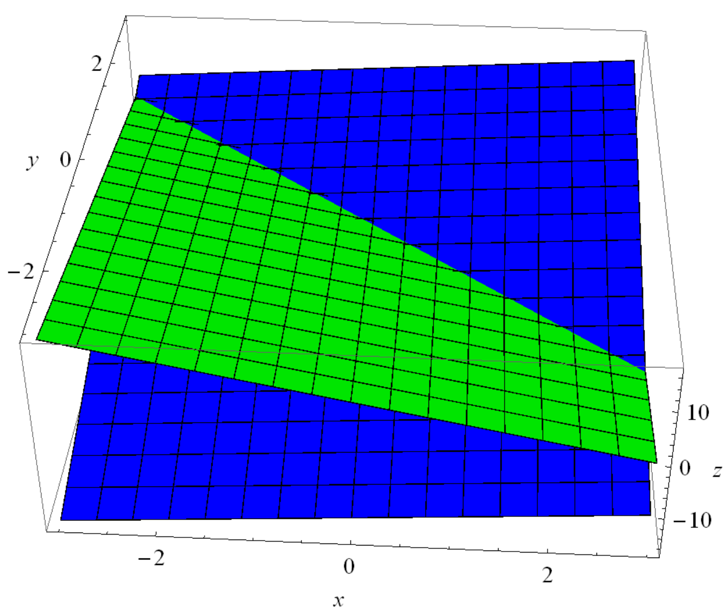
$$\mathbf{m}=(3,2,1)\times(1,4,-1)=\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ 3 & 2 & 1\\ 1 & 4 & -1\end{vmatrix}=(-2-4)\hat{\mathbf{x}}-(-3-1)\hat{\mathbf{y}}+(12-2)\hat{\mathbf{z}}=-6\hat{\mathbf{x}}+4\hat{\mathbf{y}}+10\hat{\mathbf{z}}=(-6,4,10)$$

All that's left is to find a point common to both planes.

$$\left.\begin{aligned}3(x-1)+2y+(z+1)=0\\ (x-1)+4y-(z+1)=0\end{aligned}\right\}$$

Choose $x=1$, $y=0$, and $z=-1$, for example. Then $\mathbf{b}=(1,0,-1)$, and the line is

$$\begin{aligned}\mathbf{y}(t)&=\mathbf{m}t+\mathbf{b}\\ &=(-6,4,10)t+(1,0,-1)\\ &=(-6t+1,4t,10t-1).\end{aligned}$$



In green is $3(x-1)+2y+(z+1)=0$, and in blue is $(x-1)+4y-(z+1)=0$.